Ivashin A.V.

Study of feasibility to explore χ_b-particles on U-70 accelerator.

1.Introduction

The aim of this work is to analyze the feasibility to study χ-levels Υ-particle on U-70 accelerator by investigation of following reactions $p\bar{p} \to \chi_b \to \Upsilon(1,2S)_{\to e^+e^-} + \gamma$ and $p\bar{p} \to \chi_b \to \Upsilon(1,2S)_{\to \mu^+\mu^-} + \gamma$.

Similar experiments for charmonium χ-levels have been performed in Fermilab ($E760$, $E835$ experiments) and in CERN ($R704$).

Investigation of $(b\bar{b})$ system is interesting because this states are poorly studied so far, in particular there are no measurements of full widths bottomonium χ-levels, there are no evidence for η_b-state and so on.

The interest to $(c\bar{c})$ and $(b\bar{b})$ bound states arises because quarks there are non-relativistic and these systems allows for more or less adequate theoretical treatment which, by the way, relies upon some assumptions to be tested experimentally (as an example, that asymptotical behavior with $\sqrt{s} \to +\infty$ of the χ_{b0}, χ_{b2} branchings to $p\bar{p}$ is determined by the proton electromagnetic formfactor squared).

Now a few words about of levels of $(b\bar{b})_{bound}$. Registered are 1^3S_1, 2^3S_1, 3^3S_1, 4^3S_1, 1^3P_0, 1^3P_1, 1^3P_2, 2^3P_0, 2^3P_1, 2^3P_2 levels. Predicted (see [6]) are 1^1S_0, 2^1S_0, 3^1S_0, 1^3D_1, 1^3D_2, 1^3D_3, 2^3D_1, 2^3D_2, 2^3D_3 levels.

2.Branching ratio estimation for $\chi_{b2} \to p\bar{p}$.

As we are going to study the processes $\chi_b \to \Upsilon(1,2S)_{\to e^+e^-} + \gamma$ and $\chi_b \to \Upsilon(1,2S)_{\to \mu^+\mu^-} + \gamma$, χ_{b0} is not of our interest due to its very small branching to $\Upsilon\gamma$ of order 5% in comparison with $\sim 30\%$ for χ_{b1} and χ_{b2}. The χ_{b1} and χ_{b2} decay differently. As the angular momentum of the two gluons can't be equal to 1 the χ_{b2} decays through two gluons while χ_{b1} decays through three partons (gluon and quark-antiquark pair). So the decay of the χ_{b2} is somewhat simpler (one can assume that the process $2g \to p\bar{p}$ is analogous to $q\bar{q} \to p\bar{p}$ in the reaction $e^+e^- \to p\bar{p}$).

The production of gluons occurs at a distance $1/m_b \ll 1$fm, so the gluons having energy $\sim m_b$ can be considered as quasistable particles and the full decay width of $\Gamma((q\bar{q})_{bound} \to p\bar{p})$ is

$$\Gamma((q\bar{q})_{bound} \to p\bar{p}) = \Gamma_{q\bar{q}\to 2g} w_{2g\to p\bar{p}} = \Gamma_{full} w_{2g\to p\bar{p}}, \tag{1}$$

$$B(q\bar{q})_{bound} \to p\bar{p}) = w_{2g\to p\bar{p}}, \tag{2}$$

where $w_{2g\to p\bar{p}}$ depends on mass $\sqrt{s}$ of bound state.

Now let us consider process $e^+e^- \to p\bar{p}$, which can be considered as two-stage process $e^+e^- \to q\bar{q}$ (fig.1) and $q\bar{q} \to p\bar{p}$.

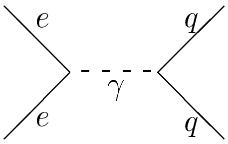

Fig.1

Matrix element of $e^+e^- \to p\bar{p}$ is

$$M_{fi}(e^+e^- \to p\bar{p}) = M_{fi}(e^+e^- \to p\bar{p})_{point} \times f(s) \tag{3}$$

where $f(s)$ – the proton electromagnetic formfactor (the $\sigma^{\mu\nu}$ term in the proton vertex is neglected). As s is constant the total cross section is

$$\sigma_{e^+e^- \to p\bar{p}} = \sigma^{point}_{e^+e^- \to p\bar{p}} \times f^2(s). \tag{4}$$

For $s \gg m_p^2 \quad \sigma^{point}_{e^+e^- \to p\bar{p}} \propto \sigma_{e^+e^- \to q\bar{q}}$, that is

$$\sigma_{e^+e^- \to p\bar{p}} = \sigma_{e^+e^- \to q\bar{q}} \times f^2(s). \tag{5}$$

It is seen from this that the transition probability for two quarks into proton and antiproton is

$$w_{q\bar{q} \to p\bar{p}} \propto f^2(s). \tag{6}$$

Making an assumption that $w_{q\bar{q} \to p\bar{p}} = w_{2g \to p\bar{p}}$ we get

$$B(q\bar{q})_{bound} \to p\bar{p}) \propto f^2_{em}(s). \tag{7}$$

Using a dipole formula for $f_{em}(s)$ we get

$$\frac{B(\chi_{b2} \to p\bar{p})}{B(\chi_{c2} \to p\bar{p})} \approx \left(\frac{M_{\chi_c}}{M_{\chi_b}}\right)^8 \tag{8}$$

$$B(\chi_{b2} \to p\bar{p}) = \left(\frac{3.5}{10}\right)^8 \cdot 10^{-4} = 2 \cdot 10^{-8} \tag{9}$$

Within used approximation we have

$$B(\chi_{b2}(1P) \to p\bar{p}) = B(\chi_{b2}(2P) \to p\bar{p}).$$

In general not branchings but the ratios of the decay width into pp to full hadronic width excluding interlevel transitions enter to the (8). But for the χ-levels the contribution of the electromagnetic decays is negligible, and the branchings of the interlevel transitions $\chi_c \to J/\psi\gamma$ and $\chi_b \to \Upsilon\gamma$ are 15% and 20% respectively. So the (8) is near correct.

3.Branchind ratio estimation for $\Upsilon \to p\bar{p}$.

Although the Υ production in $p\bar{p}$-collision has not a big interest, if we suppose power-like dependence of probability transition of 3 gluons to $p\bar{p}$, the available experimental data for charmonium allows the estimation of α in this dependence

$$w_{3g\to p\bar{p}} \propto \frac{1}{M^\alpha},$$

This dependence can take place in process $\chi_{b1} \to p\bar{p}$ due to three-parton $(q, \bar{q}, g)$ intermediate state.

So, we assume that

$$B((c\bar{c})_S \to p\bar{p}) \propto \frac{1}{M^\alpha} \cdot V_{phase},$$

where B is ratio of decay width to $p\bar{p}$ to full width of Γ_1 *excluding interlevel transitions and leptonic decays.* For J/ψ B is simply branching ratio divided by 0.7, and for ψ' Γ_1 one can calculate noticing that in non-relativistic theory $B((c\bar{c})_S \to e^+e^-) = const$, so

$$\Gamma_1(\psi') = 0.7 \cdot \frac{\Gamma(\psi' \to e^+e^-)}{B(J/\psi \to e^+e^-)}.$$

Numerical values are:

$$B(J/\psi \to p\bar{p}) = 3 \cdot 10^{-3} \pm 10\%,$$

$$B(\psi' \to p\bar{p}) = 2.3 \cdot 10^{-3} \pm 30\%.$$

For α we have expression:

$$1.1 \cdot \left(\frac{M(J/\psi)}{M(\psi')}\right)^\alpha = 0.77 \pm 30\%,$$

where 1.1 arise due to phase space.

Finally

$$\alpha = \frac{\ln 2}{\ln(3.1/3.7)} \pm 85\% = 2 \pm 85\%.$$

Worst case corresponds to $\alpha = 4$.

If for χ_{b1} α really equals to 4, than it increases the decay width of the $\chi_{b1} \to p\bar{p}$ by 2 orders in comparison with dipole formula estimation.

4.Required luminocity estimation.

Let us estimate the cross section of χ_{b2} formation in $p\bar{p}$-collision. According to Breit-Wigner formula

$$\sigma = \frac{(2J+1)}{(2S_1+1)(2S_2+1)} \frac{4\pi}{s} \frac{\Gamma_{in}\Gamma_{out}}{(E-E_0)^2 + \Gamma^2/4}, \tag{10}$$

where S_1, S_2 – proton's and antiproton's spins, J – the resonance spin.
At the resonance we have:

$$\sigma_0 = (2J+1)\frac{4\pi}{M^2}B_{in}B_{out} \tag{11}$$

Taking into account the relation $1\text{GeV}^{-1} = 0.2\text{fm}$ we get for the χ_{b2}:

$$\sigma_0 = 250 \times B_{\chi_{b2}\to p\bar{p}} \times B_{\chi_{b2}\to\gamma\Upsilon(1,2S)}\mu bn$$

$$\sigma_0 = 250 \cdot 2 \cdot 10^{-8} \cdot 0.2 = 10^{-6}\mu bn \tag{12}$$

If we want to investigate the processes

$$p\bar{p} \to \chi_b \to \Upsilon(1,2S)_{\to e^+e^-} + \gamma$$

and

$$p\bar{p} \to \chi_b \to \Upsilon(1,2S)_{\to \mu^+\mu^-} + \gamma,$$

then taking into account that $B(\Upsilon \to (e^+e^- \text{ и } \mu^+\mu^-)) = 5\%$, one can obtain event rate

$$N = \sigma_0 \mathfrak{L} \cdot 0.05 \quad , \quad \mathfrak{L} = \frac{N}{\sigma_0 \cdot 0.05},$$

where $\mathfrak{L}$ – luminocity.

For the desired $N = 1000$ ev/year $= 10^{-4}\text{s}^{-1}$ the required luminocity is

$$\mathfrak{L} = \frac{10^{-4}}{10^{-6} \cdot 10^{-30} \cdot 0.05} = 2 \cdot 10^{33}\text{cm}^{-2} \cdot \text{s}^{-1} \tag{13}$$

Accounting for the $p\bar{p}$ total cross section of 50 mbn at 10 GeV, the total event rate at this luminocity is

$$N_{all} = 50 \cdot 10^{-27} \cdot 2 \cdot 10^{33} \approx 10^8 \text{ s}^{-1} \tag{14}$$

5. The angular and energy distributions of the decay products of χ_b

Let us denote:

p - momentum of beam antiproton,
p_0 - momentum of proton at rest,
p_χ - momentum of χ-resonance.

From condition $(p+p_0)^2 = p_\chi^2 = M_\chi^2$ one can obtain

$$\mathcal{E}^{\bar{p}} = \frac{M_\chi^2 - 2m_p^2}{2m_p}, \tag{15}$$

$$p_{\bar{p}} = \frac{M_\chi}{2m_p}\sqrt{M_\chi^2 - 2m_p^2}, \tag{16}$$

$$\gamma \equiv \frac{\mathcal{E}_{\bar{p}+m_p}}{M_\chi} = \frac{M_\chi}{2m_p}. \tag{17}$$

The corresponding numerical values are given in table 1.

Let us assume that decays $\chi \to \gamma + \Upsilon$ and $\Upsilon \to e^+e^-$ are isotropic in center-of-mass system

In case of $\chi \to \gamma + \Upsilon$ the momentum transfer to Υ is ~ 0.8 GeV, so the angular spread of Υ is of order $0.8/50$ rad $\approx 1^\circ$. Therefore we neglect this angular spread and admit $\gamma_\Upsilon = \gamma_\chi$. The results for the e^+e^- distributions will be accurate within $\sim 1^\circ$.

The distribution upon the $x = \cos\theta$ for the massless e^- and γ is obtained through the Lorentz boost from c.m.s. of the decaying particle to the laboratory system:

$$\frac{dw}{dx} = \frac{1}{2(\gamma - x\sqrt{\gamma^2-1})^2}. \tag{18}$$

In case of e^+e^- x^+ and x^- are linked by expression

$$x^+ = \frac{2\gamma\sqrt{\gamma^2-1} - (2\gamma^2-1)x^-}{(2\gamma^2-1) - 2\gamma\sqrt{\gamma^2-1}x^-}. \tag{19}$$

Energy depends on x as

$$\frac{\mathcal{E}}{\mathcal{E}_{c.m.}} = \frac{1}{\gamma - \sqrt{\gamma^2-1}x}, \tag{20}$$

where $\mathcal{E}_{c.m.}$ – energy in c.m. system. Graphs for functions (19) and (20) are shown on fig.2,3 for $\gamma = 5.3$.

Table 1.

State	Mass, GeV	Energy of $\bar{p}$, GeV	Momentum of $\bar{p}$, GeV	γ-factor
$\chi_{b1}(1P)$	9.892	51.205	51.672	5.271
$\chi_{b2}(1P)$	9.913	51.426	51.893	5.282
$\chi_{b1}(2P)$	10.255	55.102	55.569	5.465
$\chi_{b2}(2P)$	10.269	55.255	55.722	5.472

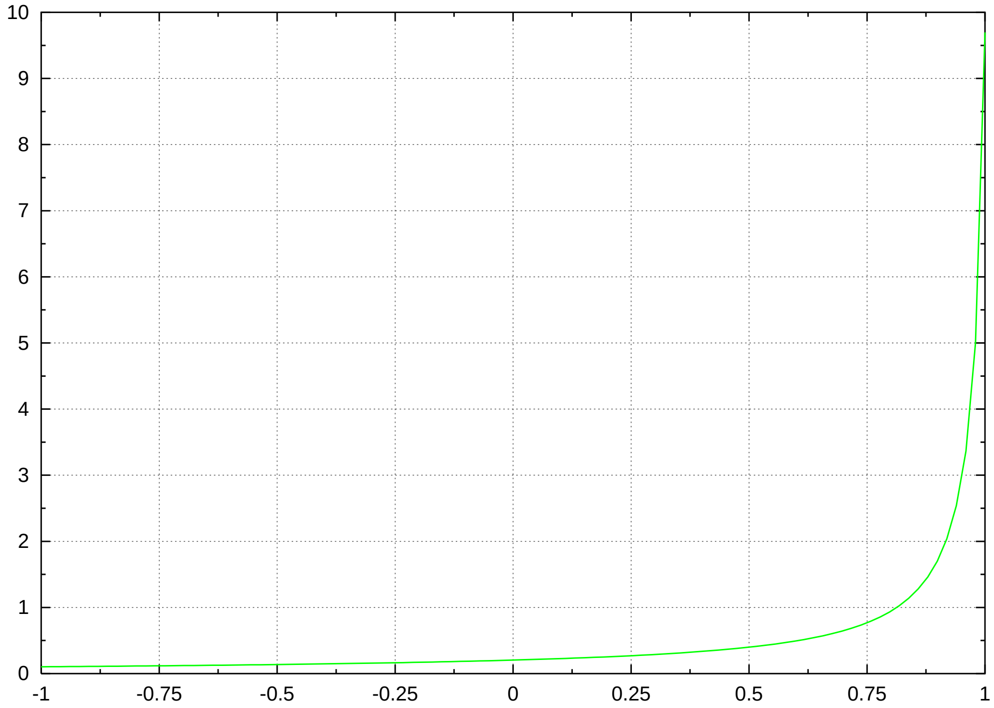

Fig.2: $\frac{\mathcal{E}(x)}{\mathcal{E}_{c.m.}}$

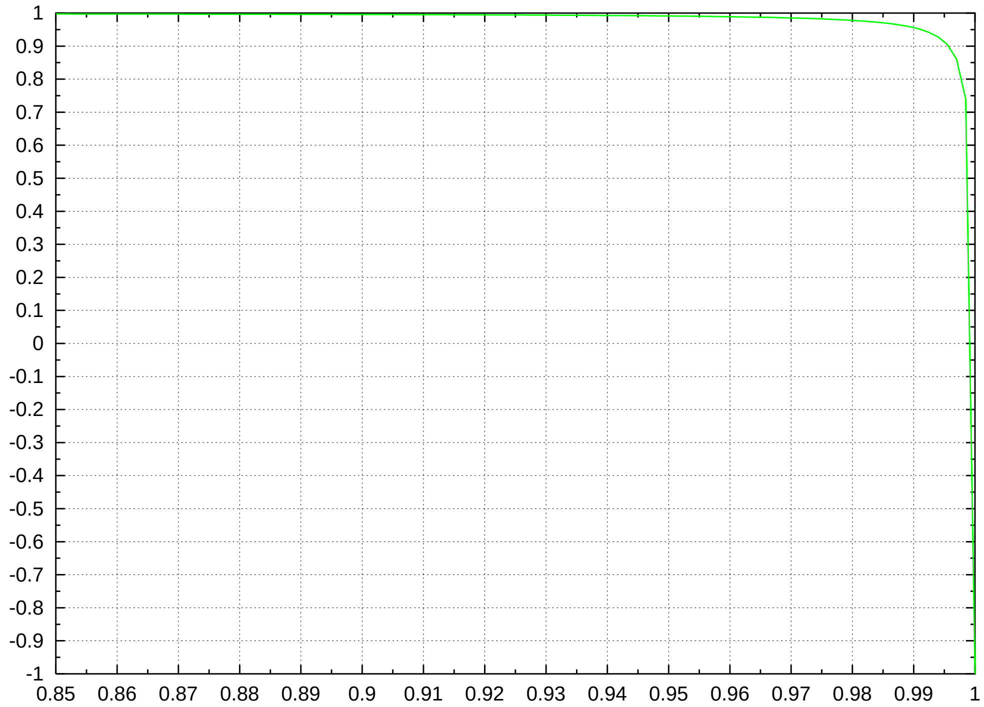

Fig.3: $x^{+}(x^{-})$.

Let us take $\gamma = 5.3$. Let $\theta_{min} = 5^\circ$, then for the leptons $\theta_{max} = f^+(\theta_{min}) = 35^\circ$ (as both leptons have to be detected), and for γ suppose $\theta_{max} = \pi/2$. It corresponds to $x \in [0.886; 0.996]$ for leptons and $x \in [0; 0.996]$ for gammas.

The acceptance for e^+e^- is equal to

$$f_e = \int_{x_{min}}^{x_{max}} \frac{dw}{dx}\, dx = \frac{1}{2\sqrt{\gamma^2-1}} \frac{1}{\gamma - x\sqrt{\gamma^2-1}} \Big|_{x_{min}}^{x_{max}} \approx 0.7 = 70\%.$$

For γ we obtain $f_\gamma \approx 85\%$. The overall geometrical efficiency equals to $f_{all} = f_e f_\gamma \approx 60\%$.

Minimal energy of γ:

160 MeV - for $x_{min} = 0$ $(f_{all} = 60\%)$,
300 MeV - for $x_{min} = 0.5$ $(f_{all} = 57\%)$,
800 MeV - for $x_{min} = 0.8$ $(f_{all} = 53\%)$.

Electron minimal energy is ≈ 7 GeV.

6.Multiplicity distribution.

Let N particles is produced as a result of a proton-antiproton collision. We suppose for the simplicity that the polar angle distributions for each particle are the same and independent. Then the probability to detect n particles is equal to $C_N^n p^n (1-p)^{N-n}$, where p is the detection probability for a single particle (the setup efficiency). The mean value for the number of the detected particles is Np. Let now N obeys the Poisson distribution. Then the probability to detect n particles is

$$p_n = \sum_N p(n|N)p(N) = \sum_{N=n}^{\infty} C_N^n p^n (1-p)^{N-n} \frac{\bar{N}^N}{N!} e^{-\bar{N}} =$$

$$= p^n \bar{N}^n e^{-\bar{N}} \sum_{N=n}^{\infty} \frac{(1-p)^{N-n}}{N!} C_N^n = \frac{(p\bar{N})^n}{n!} e^{-p\bar{N}} \qquad (21)$$

– which is again Poisson distributed.

The distribution upon transverse momentum p_t for single particle is described rather well by the formula

$$f_t(p_t) = 36 p_t e^{-6p_t}, \qquad (22)$$

where p_t – transverse momentum in GeV. The distribution upon $x \equiv p_L/p$ in center-of-mass system (p – beam momentum, p_L – particle's longitudinal momentum) for $|x| \leq 0.7$ can be described by following formula:

$$f_L(x) = 4e^{-8|x|}. \qquad (23)$$

Adopting as a first approximation the independence of p_t and x, we arrive at

$$f_{xt}(x, p_t) = f_t f_x = 144 p_t e^{-8|x|-6p_t}. \tag{24}$$

From this one can get the $\cos\theta$ disribution in laboratory system as follows:
1. Divide the range of $(-1;1)$ into N intervals and set a correspondence with a counting variable for each of them.
2. Introduce a grid on the plane (x, p_t).
3. For each node of the grid:
   1. Implement Lorentz transformation and calculate $\cos\theta$ in lab-system.
   2. Increase the counting variable of the corresponding interval by $f_{xt}(x, p_t)$

After that one can calculate the probability to register a particle within the range $\theta \in (5°; 35°)$ ($\cos\theta_{lab} \in (0.886; 0.996)$). The calculations give (for $\pi^{\pm}$) the probability $p \approx 40\%$, the mean energy in c.m.s ~ 1 GeV. To get a reliable result it is necessary that at least few points fall into each interval, the variation of f_{xt} over the grid cell has to be small, the interval width has to be much less than the range Δx over which the probability is sought for. The $\cos\theta_{lab}$ distribution is shown on fig.4.

Mean multiplicity ([1]) in $p\bar{p}$-collision at $\sqrt{s} = 10$ GeV is equal to $\bar{N} = 5$. From that $p\bar{N} = 2$, and using (21) obtain

$$w_n = \frac{2^n e^{-2}}{n!}, \tag{25}$$

where w_n – probability of registartion of n particles. Numerical values are

$w_0 = 13.5\%, \quad w_2 = 27.0\%,$
$w_1 = 27.0\%, \quad w_3 = 18.0\%.$

So if we adjust first level trigger to select 3-particles events, then we get ~ 5 background suppression. It is valid if one can neglect the multiple collisions.

Earlier the lower limit for the luminosity $2 \cdot 10^{33}$cm^{-2}·s^{-1} was obtained. However the collision rate 100 MHz is too high. Let's take more realistic value of 30 MHz, that is one collision in 30 ns ($\mathcal{L} = 6 \cdot 10^{32}$ cm^{-2}·s^{-1}). Taking into account all the losses it yields the detection of ~ 10 events of the χ_{b2} decay per year. However, the formfactor dependence $\propto 1/M^8$ is just a likely assumption, and for the χ_b there are grounds to suppose the dependence $\propto 1/M^4$ (chapter 3) which gives rise to the cross section increase by two orders.

The most fast plastic scintillators have a light decay time < 3 ns. So, if to implement the first level trigger on multiplicity as a (triple) coincidence

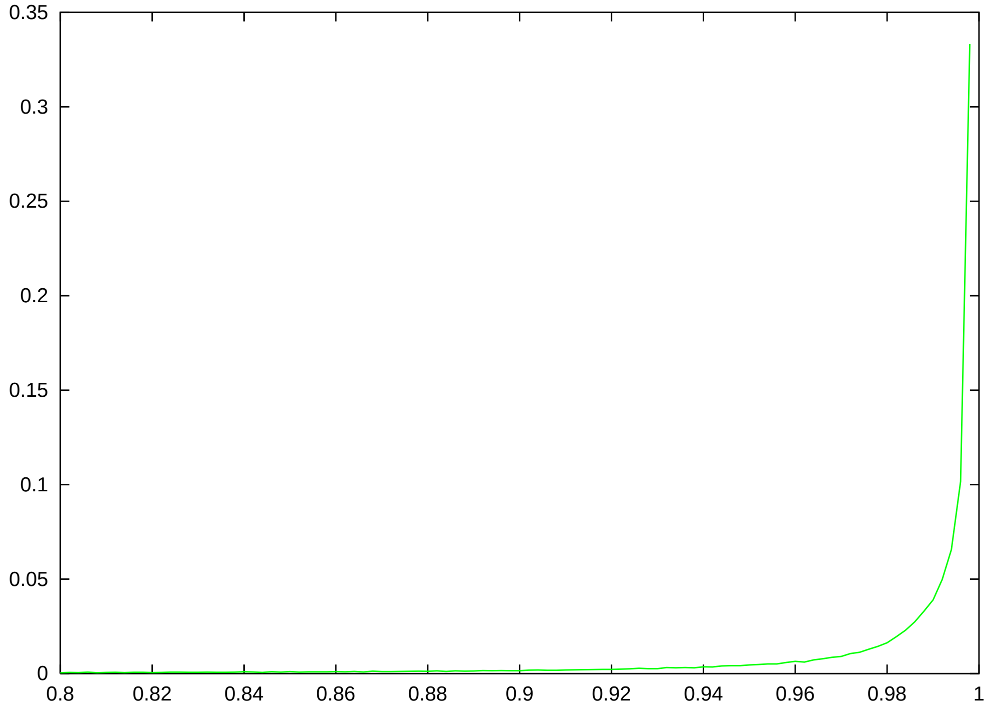

Fig.4: $\cos\theta_{lab}$ inclusive particle distribution

in $p\bar{p}$-collision.

within 3 ns, the probability of having yet another collision within this time interval is $1 - e^{-0.1} = 10\%$, that is the neglecting multiple collisions is justified. Further reduction (as the 150 ns/event is not enough for the data acquisition) can be achieved with the second level trigger on the basis of the particles identification using the transition radiation detector (TRD) for the electrons and scintillation counters behind hadron absorber (1 m of iron) for muons.

7.Hypothetical setup.

Taking into account all the aforesaid, one can sketch the required setup. It is shown in fig.5.

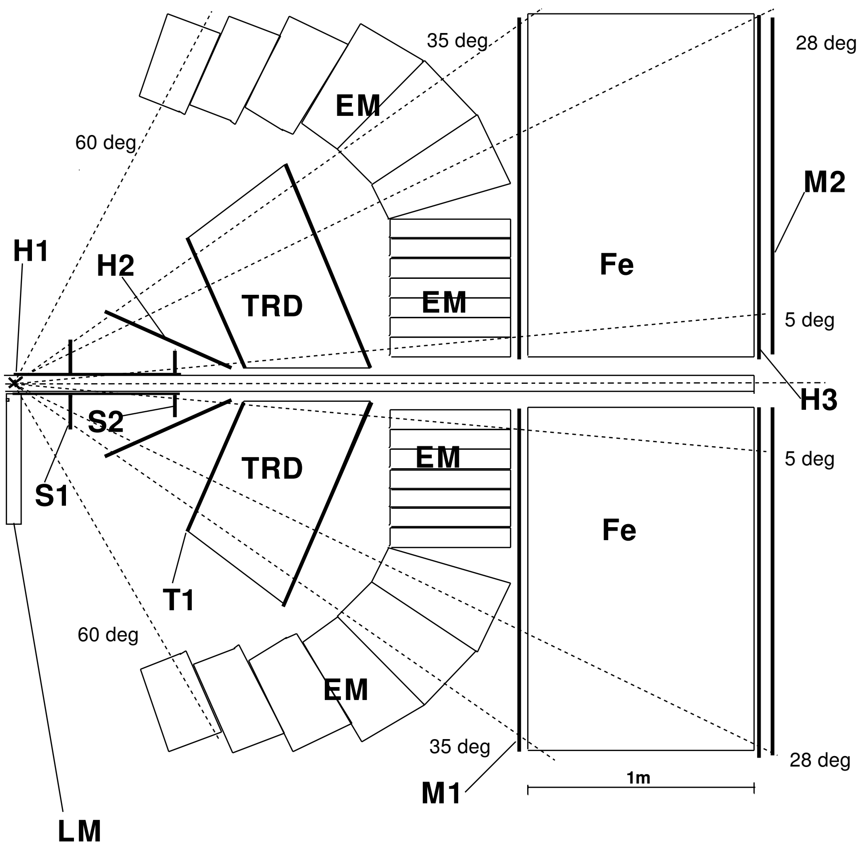

Fig.5.

Designations:

H1 , H2 , H3	– scintillation hodoscopes.
Fe	– magnetized iron.
TRD	– transition radiation detector.
T1	– tracking detectors.
S1 , S2	– silicon tracking detectors.
M1 , M2	– muon detectors.
EM	– electromagnetic calorimeter.
LM	– luminosity monitor.

The setup performance looks as follows. The low energy ~ 1 GeV antiproton accumulator is required which should store and cool down antiprotons originating from the impact of the U-70 proton beam with a fixed target. Following the storage of enough antiprotons they are injected to the U-70 ring and accelerated to the energy $\sim$ 50 GeV. The detector is mounted in a 3.7 m lengthy straight section of the U-70. The diameter of the accelerator chamber is 13 cm, the distance from the chamber center to the floor is 2.15 m, the tunnel height is 4.0 m. The interactions occure directly in the accelerator chamber at the jet hydrogen target. At the collision rate of 30 MHz the life time of the beam of $\sim 10^{13}$ antiprotons is just $\sim$ 100 hours which impose rather hard requirements to the antiprotons storage rate.

To identify the decays of the χ-particles it is supposed to calculate $\sqrt{s}$ for all "suspicious"decays and test the relations (19), (20). The momenta of muons are measured through the deflection in the magnetized iron, the energy of electrons and gammas is measured with the electromagnetic calorimeter.

The luminosity monitoring can be realized as in the E760 experiment ([3]), detecting the recoil proton in the elastic $p\bar{p}$ scattering at large ($\sim 90°$) angle to the setup axis.

Bibliography

1.C.Caso et al., The European Physical Journal, **C3**, 1, 1998, "Review of Particle Physics".

2.A.I.Vainstein, M.V.Voloshin , Usp.Fiz.Nauk **123**, 217–255, 1977, "Charmonium and QCD".

3.T.A.Armstrong et al.(E760 collaboration), Fermilab preprint Pub-91/213-E, "Study of the χ_{c1} and χ_{c2} charmonium states formed in $\bar{p}p$ annihilations".

4.T.A.Armstrong et al.(E760 collaboration), Fermilab preprint Pub-91/299-E, "Precision measurements of charmonium states formed in $\bar{p}p$ annihilation".

5.P.D.B.Collins, A.D.Martin, "Hadron Interactions", *Adam Hilger ltd, Bristol, 1984.*

6.D.Ebert, R.N.Faustov, V.O.Galkin, hep-ph/0110190.